NPS-OR-10-005

NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

A Multistatic Sonobuoy Theory

by

Alan R. Washburn

August 2010

Approved for public release; distribution is unlimited

Prepared for: Operations Research Department
Naval Postgraduate School
Monterey, CA 93943

THIS PAGE INTENTIONALLY LEFT BLANK

NAVAL POSTGRADUATE SCHOOL
MONTEREY, CA 93943-5001

Daniel T. Oliver
President

Leonard A. Ferrari
Executive Vice President and
Provost

This report was prepared for the Operations Research Department, Naval Postgraduate School, Monterey, California and partially funded by the Chair of Undersea Warfare, Naval Postgraduate School, Monterey, California.

This report was prepared by:

ALAN R. WASHBURN
Distinguished Professor Emeritus of
Operations Research

Reviewed by:

R. KEVIN WOOD
Associate Chairman for Research
Department of Operations Research

ROBERT F. DELL
Chairman
Department of Operations Research

Released by:

KARL VAN BIBBER
Vice President and
Dean of Research

THIS PAGE INTENTIONALLY LEFT BLANK

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)* 08-2010	**2. REPORT TYPE** Technical Report	**3. DATES COVERED** *(From - To)*
4. TITLE AND SUBTITLE A Multistatic Sonobuoy Theory		**5a. CONTRACT NUMBER**
		5b. GRANT NUMBER
		5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) Alan R. Washburn		**5d. PROJECT NUMBER**
		5e. TASK NUMBER
		5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000		**8. PERFORMING ORGANIZATION REPORT NUMBER** NPS-OR-10-005
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) N/A		**10. SPONSOR/MONITOR'S ACRONYM(S)**
		11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

The characterizing feature of multistatic systems is the inclusion of geographically independent receivers that might hear echoes from any source. We develop a simple analytic theory to predict detection probability, and use it to study cost/effectiveness issues and the advisability of using co-located source/receiver pairs instead of independent sources and receivers.

15. SUBJECT TERMS

Multistatic, Bistatic

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF**	**18. NUMBER OF**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT Unclassified	**b. ABSTRACT** Unclassified	**c. THIS PAGE** Unclassified	UU	32	**19b. TELEPHONE NUMBER** *(include area code)*

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18

THIS PAGE INTENTIONALLY LEFT BLANK

ABSTRACT

The characterizing feature of multistatic systems is the inclusion of geographically independent receivers that might hear echoes from any source. We develop a simple analytic theory to predict detection probability, and use it to study cost/effectiveness issues and the advisability of using colocated source/receiver pairs instead of independent sources and receivers.

THIS PAGE INTENTIONALLY LEFT BLANK

1. INTRODUCTION

We consider underwater sonar systems composed of sources and receivers. Detection occurs when a source generates a brief pulse of underwater sound that is reflected by its submarine target and then detected by one or more receivers, thereby revealing the submarine's location. The source of sound might be a ship, a helicopter with a dipping sonar, an explosive charge dropped by an aircraft, or an active sonobuoy. If the receiver is always colocated with the source, the system is called monostatic, and can be roughly characterized by the distance at which detection is barely possible, the "detection range". In multistatic systems, the sources and receivers are not colocated. Multistatic systems are more difficult to characterize than monostatic systems, but have some arguments in their favor. Among them are

- Receivers are less expensive than sources, so it makes sense to employ more receivers than sources. For example, the U.S. Navy's new SSQ-125 active sonobuoy is expected to initially cost about seven times as much as the SSQ-110 passive sonobuoy that will listen for its signals (J. Cardarelli, personal communication, February 2010).
- A multistatic system can employ different platforms for sources and receivers. A ship might be the source, while the receivers are sonobuoys.
- Sources reveal their locations to submarines when they transmit, and submarines can use that information to avoid detection. However, the independent receivers in a multistatic system do not reveal their positions.
- It is possible that the reflected signal will be received by multiple receivers. The resulting location estimate will be more precise, and the phenomenon can also help to eliminate some of the false alarms that monostatic active systems are normally prey to. Coon (1997) discusses the fusion of detections from multiple receivers.

Our main ambition here is to quantify the probability of detection in a multistatic sonobuoy system. The restriction to sonobuoys (hereafter simply "buoys") is because we intend to make no allowance for the effect of motion on the part of sources, receivers, or targets—all three entities are assumed to remain stationary in two-dimensional space. We will include the possibility that some of the sources are "posts", which will mean that the source has a colocated receiver. Although we will consistently refer to detection by sonobuoy systems, detection by multistatic radar systems is subject to the same mathematics.

The effectiveness of any sonobuoy pattern will depend on the geometric arrangement of the buoys, and the problem of selecting the arrangement to maximize the detection probability therefore arises. We will not consider such optimization questions, assuming instead that all buoys are simply located uniformly at random within some region. Our excuse for this is that our goals are strategic rather than tactical. Specifically, the goal is to quantify the detection probability as a function of the numbers of buoys of various types that are employed, rather than to determine exactly how they ought to be

arranged. While detection probability depends on the geometric arrangement, it depends even more strongly on resources, and the assumption of random deployment will make it possible to expose the dependence on resources in an analytically simple manner.

The U.S. Navy employs ASPECT, a tactical decision aid that is capable of predicting detection probability for multistatic sonobuoy fields. ASPECT considers geometric optimization questions, and employs a physical model that is more realistic than anything that will be employed here. However, ASPECT is a menu-driven Monte Carlo simulation that is not analytically suitable for considering resource allocation questions. Bowen and Mitnick (1999) describe the Multistatic Performance Prediction Methodology (MPPM). In MPPM, sources are arranged on a rectangular grid with fixed spacing, rather than randomly distributed as assumed here. Other multistatic models include the Multistatic Acoustic Simulation Model (MSASM, on which ASPECT is based), the Sonar Equation Modeling and Simulation Tool (SE-MAST) and the Surveillance Operational Concepts Model (SOCM). All of these models are more realistic and less tractable than the ones considered here.

Many of the figures and tables referred to below were generated using a Microsoft Excel™ workbook named *IEER.xls*. Possession of this workbook would allow the reader to vary parameters, iterate simulations, verify logic, and possibly discover mistakes made by the author, who would appreciate being notified if that is the case. The workbook can be found among the downloads at http://faculty.nps.edu/awashburn/ .

2. RANDOMLY PLACED FIELDS OF INDEPENDENT SOURCES AND RECEIVERS

2.1 Detection Probability

Our model of "randomly placed field" will throughout be a Poisson field. A Poisson field of points in *n*-dimensional Euclidean space is characterized by a single parameter λ, representing the average number of points per unit volume. Poisson fields have many desirable analytic properties, one of which is that the number of points inside any region with volume V is a Poisson random variable with mean λV. In particular, the probability of finding no points inside such a region is $\exp(-\lambda V)$, a fact that will be frequently employed below. We will also use two additional properties:

- The superposition of a Poisson field with density λ on an independent Poisson field with density μ is a Poisson field with density $\lambda + \mu$.
- If a Poisson field with density λ is "thinned" by removing each of its points independently with probability p, then the remaining points constitute a Poisson field with density $\lambda(1-p)$.

Consider, then, two independent, two-dimensional Poisson fields, one (sources) with density g and the other (receivers) with density h. Each source emits an omnidirectional sound that is reflected by a target, and the reflected energy is eventually received by each receiver. We assume that the transmission loss at a distance R is proportional to $R^{-\alpha}$ for some positive value of α. Spherical spreading corresponds to $\alpha = 2$, but other values of α will reasonably model any direct-path situation, so we refer

to this as a direct-path model. If R_1 and R_2 are the distances of a source and a receiver from the target, then the total transmission loss is $t=(R_1^{-\alpha})(R_2^{-\alpha})=(R_1R_2)^{-\alpha}$, so detection will depend on whether the product R_1R_2 is smaller than some threshold. The region where detection is possible is thus the interior of some Cassinian oval (Cox, 1989). The detection threshold depends on the source level, the target strength, the sensitivity of the receiver and the background noise level, but all of these can be combined into a single constant ρ with dimensions of length such that detection happens if and only if $R_1R_2 \le \rho^2$. Only the smallest such product need concern us, since detection by any source-receiver pair is sufficient for our purposes. We intend to quantify Q, the probability that the smallest range product is larger than ρ^2. Q is the nondetection probability, and $1 - Q$ is the detection probability.

Since the sources (receivers) form a Poisson field with density g (h), the two tail functions of interest are $P(R_1>r)=\exp(-\pi g r^2)$ and $P(R_2>r)=\exp(-\pi h r^2)$, for all $r \ge 0$. In each case, the event that the nearest distance is larger than r is the same as the event that there are no points in a circle with radius r; that is, we are employing the formula for the probability that a Poisson random variable is zero. Since R_1 and R_2 are independent, we have

$$Q=\int_0^\infty P(R_1>\frac{\rho^2}{r})f_{R_2}(r)dr=\int_0^\infty \exp(-\frac{\pi g\rho^4}{r^2})2\pi hr\exp(-\pi hr^2)dr\,. \qquad (2.1)$$

Let $s=\pi g\rho^2$ and $t=\pi h\rho^2$ be dimensionless versions of the source and receiver densities, and let $x=\pi hr^2$. Substituting x, s, and t into (2.1), we have

$$Q=\int_0^\infty \exp(-(\frac{st}{x}+x))dx = yK_1(y)\,, \qquad (2.2)$$

where $y=2\sqrt{st}$ and $K_1(y)$ is a modified Bessel function of order 1 (BESSELK(y,1) in Microsoft Excel™).

The detection probability is

$$P(y)=1-Q=1-yK_1(y)\,. \qquad (2.3)$$

The detection probability depends only on the "effort density" parameter y, which incorporates all factors of tactical relevance. Figure 1 shows $P(y)$ together with two approximations, one of which is accurate for small y and the other for large y. The two approximations are

$$P(y)\approx -\frac{y^2}{2}\ln(\frac{y}{2}) \text{ for small } y, \text{ and } P(y)\approx 1-\sqrt{\frac{\pi y}{2}}\exp(-y)(1+\frac{3}{8y}) \text{ for large } y \qquad (2.4)$$

Both approximations are taken from Abramovitz and Stegun (1964).

Now suppose that x_1 sources and x_2 receivers are randomly placed inside a region of area A', with no buoys outside of A', and that A' is immersed in a larger region A wherein a target is placed at random (Figure 7). We wish to choose A' to maximize the probability that the target is detected. For monostatic systems, the best choice of A' is to make it as large as possible (nearly all of A), since doing so makes it unlikely that the regions covered by the individual buoys will wastefully overlap. This is not necessarily

true with multistatic systems, the reason being that $P(y)$ is not a concave function when y is small, as is evident in Figure 1.

We assume that the target will not be detected if its position lies outside of A', and that a target inside of A' essentially faces two infinite Poisson fields with densities $g = x_1/A'$ and $h = x_2/A'$. The word "essentially" is carefully chosen, since the numbers of buoys within A' would be random in a Poisson field, rather than the fixed numbers x_1 and x_2, but there should be little difference in effectiveness when those numbers are large. The first assumption is pessimistic, since targets that are close to A' can still be detected, and the second is optimistic, since targets near the edge of A' do not face complete Poisson fields. Perhaps the two assumptions taken together are neutral, at least if A' is large. This possible neutrality will be tested in Section 6, but our purpose here is simply to explore the consequences of making both assumptions.

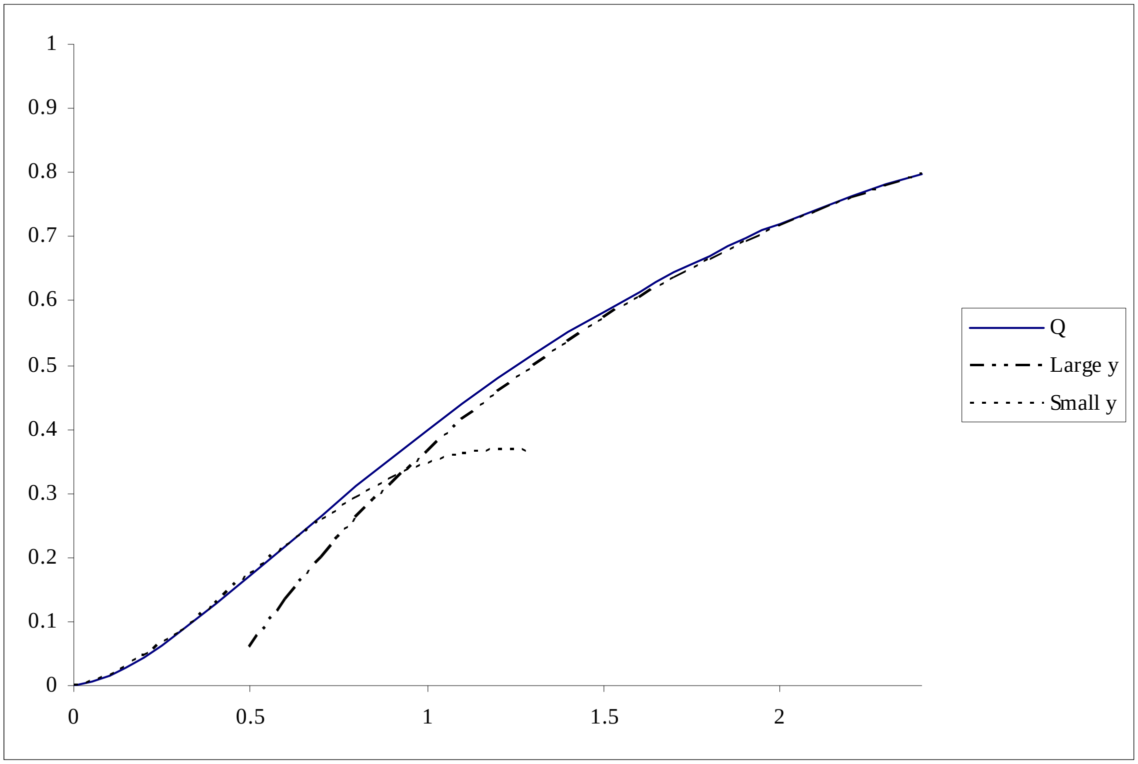


Figure 1: The detection probability *P*(*y*), together with two approximations.

The probability that the target is located in A' is A'/A, and within A' the product of the two densities is $gh = x_1 x_2 /(A')^2$, so the effort parameter within A' is $y' = \frac{2\pi\rho^2}{A'}\sqrt{x_1 x_2}$, which determines the conditional detection probability. The searcher can have a large y' in a small area or a small y' in a large area, and must decide which is best. The unconditional detection probability is

$$PD = \frac{A'}{A}P(y') = \frac{2\pi\rho^2\sqrt{x_1 x_2}}{A}\frac{P(y')}{y'} \equiv y\frac{P(y')}{y'}, \qquad (2.5)$$

where the last equality defines y. The searcher's problem is to maximize the ratio of $P(y')$ to y' by adjusting A'. Figure 2 shows a graph of the ratio. The maximizing y' is about 1.1 and the maximized ratio is almost exactly 0.4. However, y' cannot exceed y, since A' cannot exceed A. Accounting for this, the maximized PD can be obtained from (2.5). Letting the optimized detection probability be $PD^*(y)$, we have

$$PD^*(y) \equiv \begin{cases} 0.4y; \; y \le 1.1 \\ P(y); \; y \ge 1.1 \end{cases}. \tag{2.6}$$

Example: Suppose that ρ = 1km, and that the area to be searched is A = 200 km^2. Assume that x_1 = 20 sources and x_2 = 40 receivers are available, so that y = 0.89. Then the detection probability, according to (2.6), is $PD^*(0.89) = 0.36$, which is achieved by placing all buoys in a fractional (0.89/1.1) part of A.

If the number of receivers is increased to 160, then y doubles to 1.78, so the buoys should now be spread over all of A. The associated detection probability is $PD^*(1.78) = 0.67$.

If the number of receivers is instead decreased to 10, then y shrinks to 0.445, and the detection probability becomes $PD^*(0.445) = 0.18$ (exactly half of what it is when there are 40 receivers). If the buoys were mistakenly distributed evenly over all of A, instead of over the optimal fractional area, the associated detection probability would be only $P(0.445) = 0.15$.

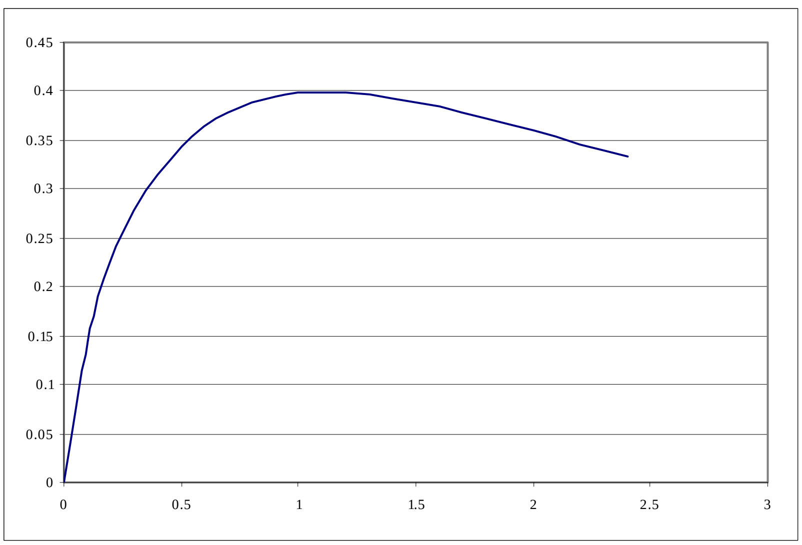


Figure 2: A graph of $P(y')/ y'$, the detection probability per unit of effort.

2.2 Equivalent Covered Area

A pattern of buoys establishes a probability of detection for a target at point (x,y), whether that point is inside or outside of the area within which the buoys are deployed. When integrated over the whole plane, that detection function becomes what might be called an "equivalent covered area" (C), with equivalency in the sense that, if a target is uniformly distributed over a region with area A that is much larger than C, then the detection probability is C/A. The question of maximizing C arises. For monostatic buoys,

the question is not interesting because the best sonobuoy field (ignoring practical questions such as how to construct or monitor widespread fields) would spread out over all of *A*. With multistatic buoys, however, the question is legitimate. The same argument as above leads to the conclusion that the effort density in the covered part of an optimal field will be about 1.1, and that the resulting equivalent area covered will be

$$C = (0.4)2\pi\rho^2\sqrt{x_1 x_2}\ . \tag{2.7}$$

We do not propose to do so here, but it would be a reasonable project to design multistatic fields that are optimal in the sense of covering the maximum equivalent area, but without our assumption that the buoys are scattered randomly. The equivalent area covered would of course be larger than (2.7).

3. SONOBUOY FIELDS THAT INCLUDE POSTS

There are some good arguments, tactical convenience being one of them, for deploying buoys in “posts” that consist of colocated sources and receivers. Cox (1989, p. 23), in response to a question about his seminal multistatic work, stated that “It makes sense to me to use multiple receivers with a single high powered source. One of these should be monostatic so that $TL_1 = TL_2$. The others are bistatic and provide increased coverage and countermeasure resistance.”

In principle, one could have three types of buoys: posts, receivers, and sources. Since we expect receivers to be cheaper than sources, however, we begin by considering fields that consist of posts and receivers, but no (independent) sources, just as Cox envisioned.

3.1 Sonobuoy Fields That Include Posts and Receivers, but No Independent Sources

3.1.1 Detection Probability

Sonobuoy fields of the type considered in this section will be called PR (Post/Receiver) fields. There is an identical theory for fields that include independent sources, but no independent receivers, since the roles of receivers and sources can always be interchanged. One of our objects is to compare the effectiveness of PR fields with that of the SR (Source/Receiver) fields that were considered above in Section 2. PR fields have no independent sources, whereas SR fields have no posts.

If any post in a PR field is closer to the target than ρ, then the miss probability is zero. Otherwise, the same logic that underlies (2.2) applies, so the formula for miss probability in a PR field is similar to (2.2). If *s* is the dimensionless density of posts and *t* is the dimensionless density of independent receivers, then the miss probability is $Q(s,st)$, where

$$Q(s,v) = \int_s^\infty \exp\left(-\left(\frac{v}{x} + x\right)\right)dx\ . \tag{3.1}$$

The analytic effect of changing sources into posts is to change the lower limit of the integral from 0 in (2.2) to *s* in (3.1), thus decreasing the miss probability. If there are no independent receivers ($t = 0$), the miss probability according to (3.1) is now $\exp(-s)$

instead of 1. There seems to be no way to express (3.1) in terms of commonly available functions. Even so, it is not difficult to approximate $Q(s,v)$ accurately using numerical integration.

Formula (3.1) assumes that the buoys are spread out in a constant density over the whole plane. If x_1 posts and x_2 independent receivers are spread out randomly over a large area A, then we can estimate the detection probability within A by substituting $s = \pi\rho^2 x_1 / A$ and $t = \pi\rho^2 x_2 / A$, as in Section 2. Also, as in Section 2, we might consider the possibility that the buoys should be concentrated in a smaller area A', or even in two overlapping areas, one for posts and one for independent receivers. However, the motivation for such concentration tactics is less strong here because the posts, considered by themselves, should theoretically be spread out over the entirety of A. Consequently, we will not consider the potential benefits of buoy concentration in PR fields.

3.1.2 Cost/Effectiveness

We next consider the question of how a given budget should be divided between sources and receivers, as well as the question of whether PR fields are superior to SR fields.

First, consider the SR case. If sources and receivers cost c_1 and c_2 each, the cost of a buoy field consisting of x_1 sources and x_2 receivers is $c_1 x_1 + c_2 x_2$. Since the detection probability is determined by the product $x_1 x_2$, it is a calculus exercise to conclude that sources and receivers should each consume half of the available budget. If the budget is B, the resulting product is $x_1 x_2 = \dfrac{B^2}{4c_1 c_2}$. Substituting this into (2.7), we find that $C = (0.4)\dfrac{\pi\rho^2 B}{\sqrt{c_1 c_2}}$. If A is the area to be covered, we can also compute $y = \dfrac{\pi\rho^2 B}{A\sqrt{c_1 c_2}}$, from which the detection probability can be computed via (2.3), or possibly via (2.6) if buoy concentration is considered.

In the special case where $c_1 = c_2$, the number of sources in an SR field should equal the number of receivers. If, instead, we were to deploy a PR field consisting purely of posts that cost $2c_1$ each, then each post would cover an area of $\pi\rho^2$, and the total area covered would be $\dfrac{\pi\rho^2 B}{2c_1}$. This exceeds the SR coverage by 20%, so we have at least one situation (small budget, equally expensive buoys) where a PR field is preferred to an SR field. A PR field might be even more effective if it included some independent receivers, a possibility that we consider next.

For subsequent investigations, we impose the cost constraint $s + ct \le b$. There is no loss of generality in restricting the coefficient of s to be 1. Suppose instead that each source costs c_1 and each receiver costs c_2, as before, with x_1 and x_2 being the numbers of sources and receivers used to cover an area of size A. If B is the total budget for the sonobuoy field, the cost constraint would be $c_1 x_1 + c_2 x_2 \le B$, which is equivalent to

$c_1 \frac{x_1 \pi \rho^2}{A} + c_2 \frac{x_2 \pi \rho^2}{A} \le \frac{B\pi\rho^2}{A}$. If we now divide through by c_1 and substitute the dimensionless densities *s* and *t* for the ratios involving x_1 and x_2, we have a constraint of the proposed form with $c = \frac{c_2}{c_1}$ and $b = \frac{B\pi\rho^2}{c_1 A}$. Using this condensed notation, we can summarize the optimal balance between sources and receivers in an SR field by

$$s = \frac{b}{2}, \quad t = \frac{b}{2c}, \quad st = \frac{b^2}{4c}, \text{ and } y = 2\sqrt{st} = \frac{b}{\sqrt{c}} \tag{3.2}$$

In a PR field, if the cost of each post is the sum of the costs of one source and one receiver, then the proper cost constraint with *c* and *b* as defined above is $(1+c)s + ct \le b$. Let $Qmin(b,c)$ be the result of minimizing $Q(s,st)$ subject to that constraint. For sufficiently small budgets, the result of this minimization will be that $t = 0$; that is, no independent receivers should be utilized. To prove this, let $Q_1(s,t)$ and $Q_2(s,t)$ be the derivatives of $Q(s,st)$ with respect to *s* and *t*, respectively, and define the function

$$H(s,t) = s\int_s^\infty \exp(-(x + \frac{st}{x}))\frac{dx}{x}. \tag{3.3}$$

Then $Q_2(s,t) = -H(s,t)$ and $Q_1(s,t) = -\frac{t}{s}H(s,t) - \exp(-(s+t))$, as can be verified by differentiating $Q(s,t)$ with respect to its two arguments. For $(s,0)$ to be minimizing, it is necessary that there exist a Lagrange multiplier λ such that $(s,0)$ minimizes the expression $Q(s,st) + \lambda(s(1+c)+tc)$, while simultaneously $s = b/(1+c)$. The first-order minimization conditions are that $Q_1(s,0) + \lambda(1+c) = 0$ and $Q_2(s,0) + \lambda c \ge 0$. Since $Q_1(s,0) = -\exp(-s)$, the first equation requires $\lambda = \exp(-s)/(1+c)$. Substituting this into the inequality, we must have $\exp(s)H(s,0) \le c/(1+c)$. Let $H(s)$ be the left-hand side of this inequality:

$$H(s) = \exp(s)\int_s^\infty \exp(-x)\frac{dx}{x} = \exp(s)[\int_0^s \frac{1-\exp(-u)}{u}du - \ln(u) - \gamma], \tag{3.4}$$

where γ is Euler's constant (0.577 …). Figure 3 shows this function.

If the budget is *b*, and if $H(b/(1+c))$ is smaller than the cost ratio $c/(1+c)$, then the entire budget should be spent on posts. It is still possible that an SR field might be better than even the best PR field, so in general we have

$$Q\min(b,c) = \min\{\exp(-\frac{b}{1+c}), PD^*(\frac{b}{\sqrt{c}})\}, \text{ for small } b. \tag{3.5}$$

Example: Consider the case $(b,c) = (1,1)$. Since H(0.5) = 0.461, which is smaller than 0.5, the best PR field will have no independent receivers. However, it is the second term in (3.5) that is minimizing, so the best field is actually of type SR. If *b* is reduced while *c* remains 1, the best PR field will still contain only posts, but will also be superior to any SR field, as in the earlier example where covered areas were compared.

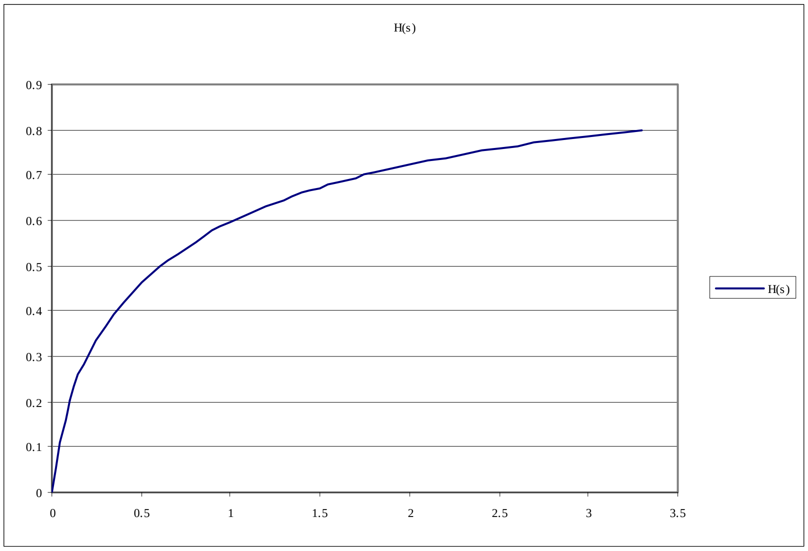


Figure 3: The function *H*(*s*) is continuous, concave, and increasing, with *H*(0)=0.

Except for small budgets, we have no analytic expression for $Qmin(b,c)$, and our only additional observation is that the optimal *s* in a PR field will always be at least as large as $\frac{b}{2(1+c)}$. This is because that value maximizes the *st* product, and *Q*(*s*,*v*) is a decreasing function of its first argument.

Example: Figure 4 shows a typical plot of *Q*(*s*,*st*) versus *s* when (*b*,*c*)=(1, 0.4). The graph demonstrates that $Qmin(1,0.4)=0.43$. The optimal value of *s* is about 0.47, which exceeds the value (0.36) that maximizes the *st* product. H(1/1.4) is about 0.5, which exceeds *c*/(1+*c*), so it should not be surprising that independent receivers are utilized in this example, as well as posts.

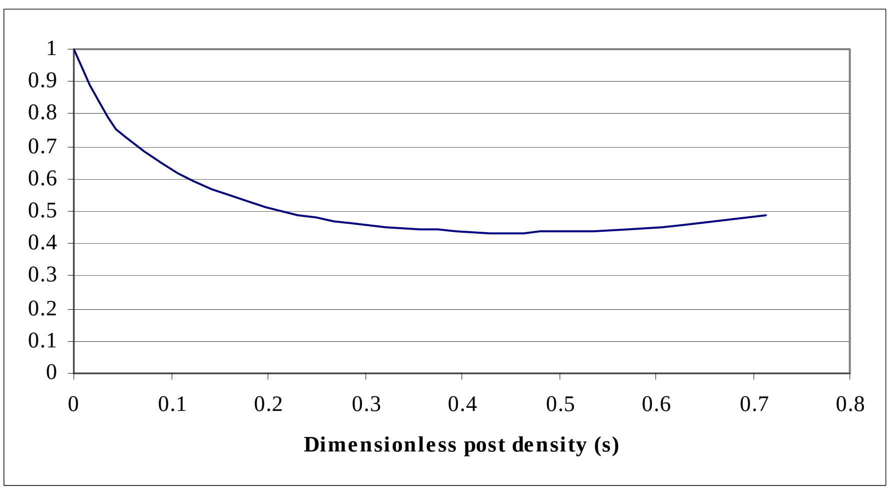


Figure 4: PR miss probability as a function of *s* when *b*=1 and *c*=0.4.

Figure 5 shows the miss probability for the best SR and PR fields when all buoys are equally expensive, with PR winning for small budgets. The SR curve is based on (2.6) with $y = b$ (y and b are equal when $c = 1$), while the PR curve is based on (3.1). The initial slope of the SR curve is –0.4. The initial part of the PR curve employs no receivers, and is consequently equal to $\exp(-b/(1+c))$, the initial slope of which is –0.5. This comparison of initial slopes explains why PR is superior when the budget is small and all buoys are equally expensive. The initial slope of the SR curve is, in general, $-0.4/\sqrt{c}$, while the initial slope of the PR curve is $-1/(1+c)$. By comparing the two expressions, it is not difficult to show that the PR field will be superior for small budgets as long as $0.25 \le c \le 4$. For more extreme values of c, the SR field is superior for all budgets. The SR field is superior for large budgets even when $c = 1$, as Figure 5 makes clear. Depending on circumstances, either type of field can be superior. If the SSQ-125 active buoy turns out to be as expensive as estimated in the introduction ($c = 1/7$), then, regardless of budget, an SR field will be more cost/effective than a PR field.

Lack of universal dominance of either the PR or the SR type suggests that the optimal sonobuoy field might actually include all three of the fundamental buoy types: posts, free receivers, and free sources. This possibility is not covered by any of the above developments, so we examine it in the next section.

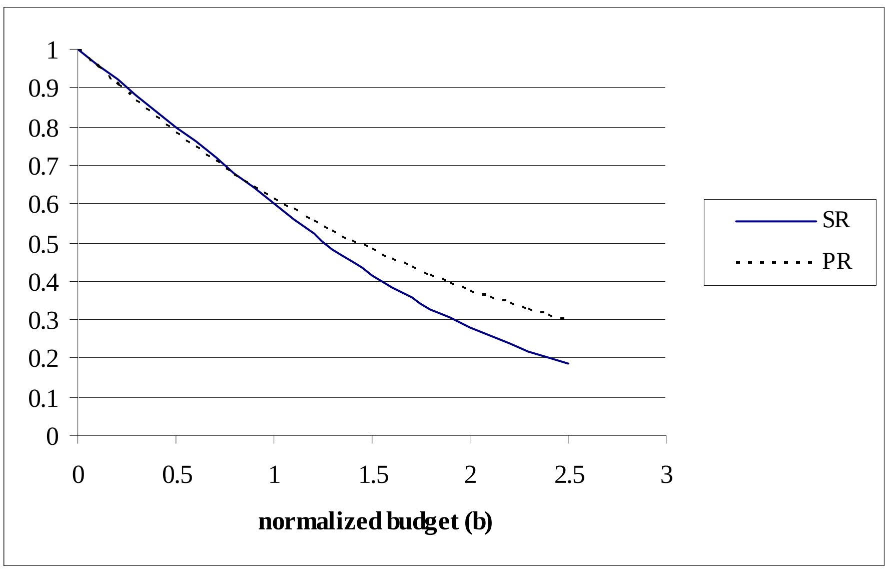


Figure 5: Miss probability for SR and PR fields when sources and receivers have the same cost (c=1).

3.2 Sonobuoy Fields That Include Posts, Independent Receivers, and Independent Sources

3.2.1 Detection Probability

In the general case, the density of posts is f, the density of free sources is g, and the density of free receivers is h. Let R_f, R_g, and R_h be the distances from the target to the nearest post, free source, and free receiver, respectively. The nondetection event can be decomposed into two mutually exclusive parts E_1 and E_2, where

$$E_1 = (R_g < \rho), (R_f > \rho), (R_f R_g > \rho^2), (R_g R_h > \rho^2), (R_f R_h > \rho^2)$$
$$E_2 = (R_g > \rho), (R_f > \rho), (R_f R_g > \rho^2), (R_g R_h > \rho^2), (R_f R_h > \rho^2) \tag{3.6}$$

The commas separating events in (3.6) should be read "and". The target will not be detected if and only if there is no post within ρ of the target, and if every product of two ranges exceeds ρ^2. These are the last four events in both E_1 and E_2, and the first event simply partitions on whether R_g is small (E_1) or large (E_2) (nondetection is also possible if $R_g = \rho$, but that possibility is safely ignored in (3.6) because it has probability zero).

In the expression for E_1 in (3.6), the second event can be omitted because it is implied by the first and third, and the fifth event can be omitted because it is implied by the first, third and fourth. In E_2, the third event can be omitted because it is implied by the first and second. Now let R_s and R_r be the distances from the target to the nearest source (free or not) and receiver (free or not). Thus, $R_s = \min(R_f, R_g)$ and $R_r = \min(R_f, R_h)$. Random variables R_s and R_r are easily described mathematically, since the posts, together with either the independent sources or independent receivers, constitute Poisson fields with densities $f+g$ (sources) and $f+h$ (receivers). Furthermore, given the above observations, E_1 and E_2 can be expressed as

$$E_1 = (R_g < \rho), (R_r R_g > \rho^2) \text{ and } E_2 = (R_s > \rho), (R_s R_h > \rho^2). \tag{3.7}$$

After conditioning on R_g in E_1 and R_s in E_2, we have

$$P(E_1) = \int_0^\rho 2\pi x g \exp(-\pi g x^2) \exp(-\pi (f+h) \frac{\rho^4}{x^2}) dx, \text{ and} \tag{3.8}$$

$$P(E_2) = \int_\rho^\infty 2\pi x (f+g) \exp(-\pi (f+g) x^2) \exp(-\pi h \frac{\rho^4}{x^2}) dx. \tag{3.9}$$

Now, let $s = \pi f \rho^2$, $t = \pi g \rho^2$, and $z = \pi h \rho^2$ be the dimensionless densities of posts, sources, and receivers, respectively, and substitute $u = \pi g x^2$ into (3.8). We then have

$$P(E_1) = \int_0^t \exp(-(u + \frac{s+z}{u})) du. \tag{3.10}$$

Similarly, substituting $u = \pi(f+g)x^2$ into (3.9), we have

$$P(E_2) = \int_{s+t}^\infty \exp(-(u + \frac{z(s+t)}{u})) du = Q(s+t, z(s+t)), \tag{3.11}$$

where Q() is the function defined in (3.1). Define the sum of (3.10) and (3.11) to be $Q(s, t, z)$:

$$Q(s,t,z) = \int_0^t \exp(-(u + \frac{s+z}{u})) du + \int_{s+t}^\infty \exp(-(u + \frac{z(s+t)}{u})) du. \tag{3.12}$$

This function represents the miss probability when a target is subjected to multistatic detections by posts, sources, and receivers in the dimensionless densities s, t, and z, respectively. The arguments t and z can be interchanged, although the form of (3.12) does not make that apparent.

3.2.2 Optimization

While we have no proof of the fact, it appears from extensive experimentation that the best sonobuoy field never includes all three types, whatever the budget or the costs of sources and receivers. Given x_1 sources and x_2 receivers, together with the ability to deploy any pair as a post, the best sonobuoy field will always have as many posts as possible (a PR field) or none at all. Except in establishing this conclusion, formula (3.12) appears to have little use.

4. RELIABILITY AND DIRECT BLAST

The idea that detection is merely a matter of being close enough to the target underlies all of the above calculations, but is only approximately true in the real world. One reason for this is that some buoys may not function correctly, in which case their distances from the target are immaterial. This possibility is easily handled theoretically in SR fields. If r_s and r_r are the reliabilities of sources and receivers, it is simply a matter of replacing the dimensionless densities s and t by sr_s and tr_r, respectively. This is because Poisson fields "thinned" in this manner remains Poisson fields. A similar argument will not work for PR fields because a post might get effectively turned into an independent source if its receiver should not function.

Proximity might not be sufficient even when all buoys function as intended. In multistatic sonar systems, receivers hear the transmitted signal directly, in addition to the signal reflected from the target. This "direct blast" is actually necessary for locating the target because the difference in time between the direct and reflected signals establishes an ellipse upon which the target must lie. However, the direct blast is much stronger than the reflected signal, and may completely obscure it if the time difference in arrivals is small enough. The principal situation where this happens is when the target is more or less directly between the source and the receiver. A simple approximation of the effect is the "dead zone" shown in Figure 6, where receivers will not detect the target because of the direct blast arriving nearly simultaneously. The angle θ measures the extent of the pie-shaped dead zone, with $\theta = \pi$ corresponding to no dead zone at all.

It is not difficult to include the effect of dead zones in a Monte Carlo simulation that begins by simulating the locations of the target, the sources and the receivers. For each source/receiver pair that passes the proximity test, one simply checks whether the receiver is in the dead zone of the source, declaring the detection attempt to be a failure if so. The resulting simulation is only slightly more complicated than one without dead zones. Unfortunately, however, this simplicity does not extend to analytic attempts to find a generalization of (2.3). We are not aware of any exact formulas for detection probability, but the following theorems at least offer lower bounds.

Theorem 1: In an SR field, let s and t be the dimensionless densities of sources and receivers, respectively, as in Section 2, but add the requirement that the receiver buoy in a successful pair must not be in the dead zone of its source. Let $p \equiv \theta / \pi$, and let $y = 2\sqrt{pst}$. If y is substituted into (2.3) one has a lower bound on the detection probability.

Proof: Let E be the event that the target is detected by the source that is closest to it. E will happen if and only if there is at least one eligible receiver that is sufficiently close to the target; a receiver being "eligible" if and only if it is not in the dead zone of the closest source. The probability that any receiver is eligible is p for each independent receiver, so eligible receivers are a Poisson field with dimensionless density pt. To calculate $P(E)$, we can now proceed as in the derivation of (2.3), except that pt needs to substituted for t. The effect of this is that y is modified as in the statement of the theorem, and $P(E) = 1 - yK_1(y)$. This is a lower bound on the detection probability because it is possible for the target to be detected even when the closest source fails. **QED**

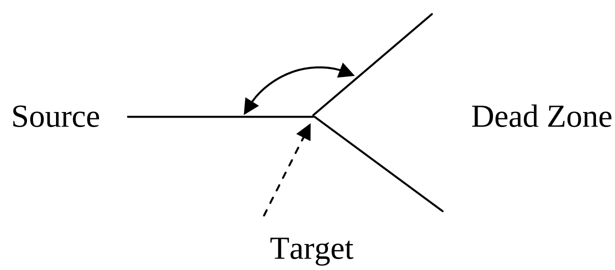


Figure 6: Illustrating a dead zone where receivers are useless on account of direct blast.

Theorem 2: In a PR field, let s and t be the dimensionless densities of posts and receivers, respectively, as in Section 3, but add the requirement that the receiver buoy in a successful pair must not be in the dead zone of its source. Let $p \equiv \theta / \pi$. Then $1 - Q(s, pst)$ is a lower bound on detection probability, where $Q()$ is the function defined in (3.1).

Proof: If the post that is nearest to the target is closer than ρ, then detection is certain because a post's own receiver cannot be in a dead zone. Let E be the event that the nearest post is farther away than ρ, but nonetheless detects the target through some other eligible independent receiver. E will happen if and only if there is at least one such receiver that is sufficiently close to the target. The probability that any independent receiver is eligible is p, so eligible receivers are a Poisson field with dimensionless density pt. To calculate $P(E)$, we can now proceed as in the derivation of (3.1), except that pt needs to substituted for t. The result of this is as stated in the theorem. This is a lower bound on the detection probability because it is possible for the target to be detected even when the closest post fails. **QED**

In both theorems, dead zones are handled as if the independent receivers had a reliability of p. Receivers that are part of a post are in effect assumed to be perfectly reliable in Theorem 2.

The question arises as to whether the upper bounds defined in Theorems 1 and 2 are sharp. We note that the bound in both theorems is exact when $\theta = \pi$ or (trivially)

when θ = 0. For intermediate values, the sharpness question will be tested by simulation in Section 6.2.

5. CAVEATS

We emphasize that all of the above analysis applies only to the direct-path transmission mode. The primary sonar modes where such a transmission loss model does *not* apply are the bottom-bounce and convergent-zone (CZ) modes. Consider the latter case, and assume that there is only one CZ at a distance R^* (typically about 50 km) with width Δ (typically about 5 km). There will be a CZ detection if and only if the circular annulus about the target at distance R^* and width Δ contains at least one buoy of each type. Since the area of this annulus is (approximately) $2\pi\Delta R^*$, the probability of this is

$$PD = (1-\exp(-2\pi\Delta R^* g))(1-\exp(-2\pi\Delta R^* h)) \approx (2\pi\Delta R^*)^2 gh\,. \qquad (5.1)$$

The approximation in (5.1) depends only on the product gh, so seemingly there is an analogy with the direct-path case. However, the approximation is actually an upper bound that is accurate only when PD is small. For sonobuoy fields that are dense enough to assure a large detection probability, the CZ case differs significantly from the direct path case.

Example: Suppose that R^*=50 km, Δ=5 km, g = 0.001/km^2, and h = 0.002/km^2. Then, using (5.1), PD = 0.76. Suppose there is also a direct path mode with ρ = 10 km. The associated y is about 0.89, and the associated direct path detection probability from (2.3) is about 0.36, significantly smaller than the CZ probability. When a CZ exists, it will generally provide most of the detection probability, as it does here. However, CZs can be ephemeral enough to make tracking difficult even in monostatic systems, and dependence on multiple CZs in the multistatic case is unlikely to improve the situation.

There are other assumptions made above that can differ significantly from the real world. Direct blast regions only approximately resemble the conical region of Figure 6. The pervasive assumptions of radial symmetry made above can be significantly false in the ocean, where transmission loss often depends on direction, as well as range. Our analysis makes no allowance for target motion, or for any phenomenon that might make ρ a function of time. Better forecasts of detection probability in a particular instance will always come from simulations that account for some of these phenomena.

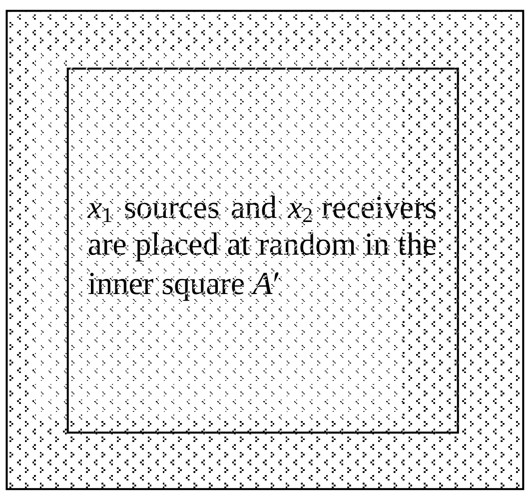
6. CALIBRATION FOR SR FIELDS

6.1 The Neutrality Assumption

In this section, we test the neutrality assumption lying behind (2.5) and (2.7). Recall that A' is the area within which the buoys are placed, while A is the potentially larger area within which targets are to be found (Figure 7). The neutrality assumption is that targets detected outside of A' will compensate for targets not detected inside of A'.

The assumption is clearly optimistic if A and A' are equal, since in that case there are no targets outside of A'. It is just as clearly pessimistic if buoys are very dense, since in that case all targets outside of A' that are within ρ of A' will be detected, in addition to all targets inside of A'. Much depends on the density of buoys and the relationship between A and A'. We begin by assuming that A is much larger than A', and that A' has been adjusted to maximize the detection probability; that is, we wish to compare the theoretical equivalent covered area C from (2.7) with an experimental version C_{exp}.

To estimate C_{exp}, without loss of generality we first take $\rho = 1$. We also take $A' = 2\pi\sqrt{x_1 x_2}/1.1$, since this is the area that theoretically maximizes detection probability. All buoys are placed within a square of side L with that area, so $L^2 = A'$. Since detection is impossible for targets more than 1 unit away from this square, we generate 30 targets at random within a square of side $L+2$ (so $A = (L+2)^2$), and count the number X that are detected. A target is detected if the product of its distances from the closest receiver and the closest source is smaller than 1. This experiment is repeated 3,000 times; that is, all the buoys are randomly relocated in A' and each such pattern is tested against 30 targets randomly located in A. Let X be the total number of targets detected in all 3,000 replications. Then the equivalent covered area is $C_{exp} = AX/90,000$.



the target is placed at random in the outer square A

Figure 7: A coordinate framework for simulation experiments.

Table 1 shows the ratio C_{exp}/C for various values of x_1 and x_2. It should be evident that the neutrality assumption is optimistic, but only by a small amount. The average of all 100 ratios is 96.6%, so the theoretical area coverage is about 3% high. Some idea of the variability of results can be obtained by comparing symmetric entries (x_1,x_2) and (x_2,x_1), since both have the same theoretical mean.

	5	10	15	20	25	30	35	40	45	50	AVG
5	0.974	0.957	0.966	0.960	0.952	0.963	0.955	0.955	0.957	0.951	0.959
10	0.946	0.976	0.959	0.972	0.969	0.961	0.955	0.965	0.958	0.965	0.963
15	0.957	0.957	0.958	0.972	0.956	0.968	0.962	0.967	0.955	0.966	0.962
20	0.963	0.953	0.968	0.965	0.972	0.963	0.972	0.963	0.965	0.962	0.965
25	0.956	0.963	0.965	0.968	0.976	0.975	0.968	0.967	0.968	0.983	0.969
30	0.958	0.964	0.963	0.971	0.967	0.963	0.976	0.964	0.977	0.971	0.968
35	0.947	0.966	0.960	0.967	0.966	0.971	0.973	0.969	0.978	0.977	0.967
40	0.960	0.969	0.971	0.958	0.972	0.974	0.968	0.972	0.976	0.969	0.969
45	0.962	0.965	0.967	0.971	0.970	0.975	0.985	0.970	0.974	0.976	0.972
50	0.952	0.972	0.967	0.978	0.974	0.975	0.969	0.971	0.973	0.978	0.971
AVG	0.957	0.964	0.965	0.968	0.967	0.969	0.968	0.966	0.968	0.970	0.966

Table 1: Rows and columns are labeled by the number of sources and receivers, respectively. The table entries are the ratio C_{exp}/C, the ratio of experimental to theoretical equivalent covered area.

We next consider the case where A and A' are both the same square, in which case we expect the theory to be optimistic. Without loss of generality, we take A' to be a unit square, and place all buoys and the target within it. The fundamental experiment is now to measure Z, the product of the distances from the target to the nearest source and the nearest receiver, recording a detection if $Z \le \rho^2$. Alternatively, let $Y = 2\pi\sqrt{x_1 x_2}Z$ and $y = 2\pi\sqrt{x_1 x_2}\rho^2$, so that detection is equivalent to the event $Y \le y$. If we now measure the cumulative distribution function of Y, we can deal with all values of ρ simultaneously. The fundamental experiment is repeated with 30 independent targets for each buoy configuration, and then for 3,000 random buoy configurations. The results are shown in Figure 8 for 20 sources and 40 receivers, where "data" (the sample cumulative distribution function or sample CDF) is compared with "theory" (formula (2.3) for $P(y)$). It can be seen that the theory is indeed optimistic in this case. The contrast would be even larger if the sample CDF were compared with (2.6) instead of (2.3), since theory would advise concentrating the buoys in a smaller area when y is small. Results for other numbers of sources and receivers are visually identical, so are not shown.

Figure 9 shows a similar comparison for a case where the targets are located in an area A that is larger than A'. Specifically, A has side of 1.2 while A' still has a side of 1, so A'/A is $1/(1.2)^2$, or 0.694. This ratio is an upper bound on the theoretical detection probability because the theoretical assumption is that targets outside of A' will not be detected. If $\rho \ge 0.1$, however, every point in A (except in the corners) is within ρ of some point in A', so a sufficiently large density of buoys within A' will detect nearly every target. Figure 9 shows that the sample CDF agrees closely with the theoretical formula (2.3) for small values of the effort density within A', but that (2.3) predicts too small a detection probability in dense fields (large values of y'). With 20 sources, 40 receivers, and $\rho = 0.1$, y' is 1.78—approximately the value at which the two curves begin to diverge.

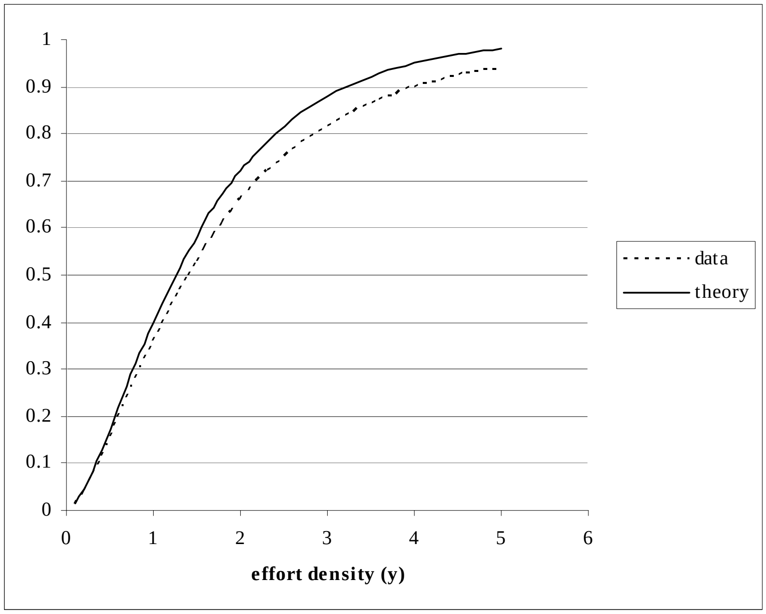


Figure 8: Comparing theoretical and experimental detection probabilities in a case where 20 sources and 40 receivers are randomly distributed over the same area that contains the target.

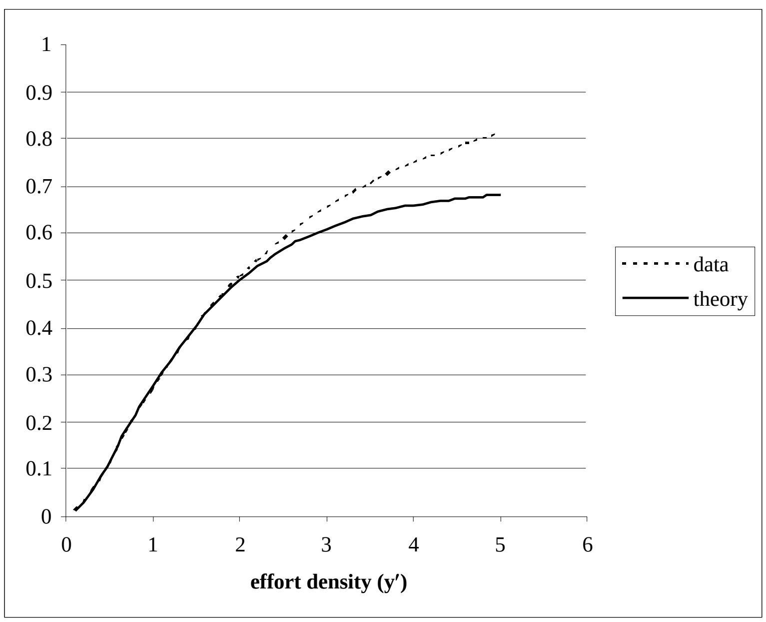


Figure 9: Comparing theoretical and experimental detection probabilities in a case where 20 sources and 40 receivers are randomly distributed in an area *A′* that is smaller than the area that contains the target. With a high effort density in *A′*, some targets outside of *A′* are detected experimentally, but not in theory.

6.2 A Direct Blast Experiment

Each number in Table 2 is the ratio of an experimentally determined effective area of coverage to the theoretical number determined from (2.7), as in Table 1. The experiment is as in Section 6.1, except that

- px_2 is substituted for x_2 in calculating the effective area covered from (2.7), as well as in calculating A', where $p = \theta/\pi$;
- in each replication, the sonobuoy field is generated independently within A' for each target; and
- an angular test is added to the proximity test before a detection is recorded, as pictured in Figure 6.

Ten thousand replications are carried out in a Monte Carlo simulation where each (source, receiver) pair of buoys is tested until some pair finally detects the target, in which case the detection counter is incremented and a new replication begun, or no detection is recorded if no pair detects the target.

The case $\theta = \pi$ is covered by Table 1, which shows that the theoretical formula (2.7) is about 3% optimistic. When $\theta = 3\pi/4$, Table 2 shows that the formula is about 5% pessimistic. When $\theta = \pi/2$, Table 3 shows that the formula is about 13% pessimistic, and when $\theta = \pi/4$, Table 4 shows that the formula is about 27% pessimistic. Presumably the increasing pessimism is because sources other than the closest source become increasingly important as the direct blast effect becomes stronger.

	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00	50.00	AVG
5.00	1.06	1.05	1.03	1.04	1.01	1.03	1.04	1.02	1.03	1.04	1.04
10.00	1.04	1.05	1.05	1.03	1.04	1.07	1.07	1.04	1.05	1.04	1.05
15.00	1.01	1.07	1.02	1.03	1.04	1.04	1.06	1.05	1.03	1.04	1.04
20.00	1.04	1.04	1.02	1.05	1.06	1.02	1.04	1.05	1.07	1.05	1.04
25.00	1.05	1.03	1.05	1.03	1.06	1.05	1.05	1.02	1.02	1.05	1.04
30.00	1.03	1.06	1.05	1.04	1.03	1.05	1.06	1.07	1.06	1.04	1.05
35.00	1.02	1.06	1.04	1.07	1.04	1.05	1.05	1.02	1.04	1.05	1.04
40.00	1.01	1.05	1.03	1.03	1.02	1.01	1.04	1.05	1.07	1.08	1.04
45.00	1.00	1.05	1.03	1.04	1.03	1.06	1.06	1.05	1.03	1.04	1.04
50.00	1.02	1.01	1.05	1.04	1.05	1.06	1.02	1.05	1.06	1.03	1.04
AVG	1.03	1.05	1.04	1.04	1.04	1.04	1.05	1.04	1.05	1.05	1.04

Table 2: Rows and columns are labeled by the number of sources and receivers, respectively. The table entries are the ratio C_{exp}/C based on 10,000 repetitions of the experiment of locating a target in an SR field of 20 sources and 40 receivers, with $\theta = 3\pi/4$.

	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00	50.00	AVG
5.00	1.13	1.14	1.10	1.15	1.10	1.12	1.14	1.10	1.14	1.12	1.12
10.00	1.14	1.14	1.18	1.12	1.11	1.12	1.12	1.12	1.12	1.12	1.13
15.00	1.12	1.09	1.15	1.12	1.16	1.12	1.15	1.13	1.14	1.15	1.13
20.00	1.15	1.13	1.13	1.12	1.11	1.13	1.13	1.13	1.13	1.09	1.13
25.00	1.10	1.12	1.11	1.14	1.12	1.13	1.16	1.14	1.13	1.15	1.13
30.00	1.09	1.13	1.14	1.13	1.18	1.14	1.15	1.14	1.15	1.14	1.14
35.00	1.09	1.11	1.18	1.16	1.12	1.12	1.13	1.11	1.15	1.16	1.13
40.00	1.10	1.11	1.13	1.12	1.14	1.14	1.16	1.15	1.15	1.14	1.13
45.00	1.11	1.09	1.14	1.14	1.13	1.14	1.15	1.13	1.16	1.13	1.13
50.00	1.08	1.10	1.11	1.13	1.14	1.17	1.13	1.14	1.16	1.12	1.13
AVG	1.11	1.12	1.14	1.13	1.13	1.13	1.14	1.13	1.14	1.13	1.13

Table 3: As in Table 2, but with $\theta = \pi/2$.

	5.00	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00	50.00	AVG
5.00	1.29	1.23	1.25	1.22	1.21	1.24	1.21	1.21	1.18	1.20	1.23
10.00	1.26	1.23	1.25	1.27	1.29	1.26	1.25	1.26	1.24	1.22	1.25
15.00	1.26	1.25	1.28	1.28	1.27	1.29	1.28	1.28	1.25	1.28	1.27
20.00	1.21	1.28	1.28	1.29	1.29	1.27	1.27	1.29	1.29	1.25	1.27
25.00	1.24	1.29	1.26	1.26	1.28	1.28	1.28	1.25	1.27	1.27	1.27
30.00	1.22	1.26	1.25	1.27	1.29	1.29	1.29	1.30	1.30	1.26	1.27
35.00	1.20	1.26	1.29	1.28	1.24	1.29	1.29	1.28	1.29	1.26	1.27
40.00	1.26	1.29	1.24	1.30	1.28	1.31	1.28	1.25	1.30	1.28	1.28
45.00	1.24	1.28	1.25	1.26	1.25	1.26	1.25	1.28	1.28	1.27	1.26
50.00	1.25	1.25	1.28	1.27	1.29	1.27	1.28	1.30	1.29	1.28	1.27
AVG	1.24	1.26	1.26	1.27	1.27	1.27	1.27	1.27	1.27	1.26	1.26

Table 4: As in Table 2, but with $\theta = \pi/4$.

7. SUMMARY

We have developed several simple formulas for approximating the detection probability of a multistatic sonobuoy field, all of which depend on the monostatic detection distance ρ. Either an SR field (no posts) or a PR field (all sources are posts) may be the most cost/effective, depending on the budget level and the relative costs of the two buoy types. If x_1 sources and x_2 receivers are used to detect a target within an area A, then the dimensionless coverage ratio,

$$y = \frac{2\pi\rho^2\sqrt{x_1 x_2}}{A},$$

is fundamental. If y is small compared to 1, then detection within A is unlikely, or the opposite is true if the ratio is large.

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF REFERENCES

Abramovitz, S., & Stegun, I. (1964). *Handbook of mathematical functions*. Applied Mathematics Series 55 (pp. 374-8). Washington, D.C.: National Bureau of Standards.

Bowen, J., & Mitnick, R. (1999). A multistatic performance prediction methodology. *JOHNS HOPKINS APL TECHNICAL DIGEST*, *20*(3).

Coon, A. (1997). Spatial correlation of detections for impulsive echo ranging sonar. *JOHNS HOPKINS APL TECHNICAL DIGEST*, *18*(1), 105-112. Accesssed in 2010 from http://www.jhuapl.edu/techdigest/td1801/coon.pdf .

Cox, H. (1989). Fundamentals of bistatic active sonar. In Y. Chan (Ed.), *Underwater acoustic data processing* (pp. 3-24). Kluwer.

THIS PAGE INTENTIONALLY LEFT BLANK

INITIAL DISTRIBUTION LIST

1. Research Office (Code 09)...1
   Naval Postgraduate School
   Monterey, CA 93943-5000

2. Dudley Knox Library (Code 013)...2
   Naval Postgraduate School
   Monterey, CA 93943-5002

3. Defense Technical Information Center 2
   8725 John J. Kingman Rd., STE 0944
   Ft. Belvoir, VA 22060-6218

4. Richard Mastowski (Technical Editor) 2
   Graduate School of Operational and Information Sciences (GSOIS)
   Naval Postgraduate School
   Monterey, CA 93943-5219

5. Distinguished Professor Alan Washburn ......………………………………….............3
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA 93943-5129

6. Professor James Eagle……………………………………………………...……….1
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA 93943-5219

7. Senior Lecturer Jeffrey Kline ……………………………………………………….1
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA 93943-5219

8. Naval Undersea Warfare Center………………………………………...……….1
   Combat Systems Department
   ATTN: Dr. Chidambar Ganesh
   Newport, RI 02841

9. Center for Naval Analyses……………………….......……………………………..1
   4825 Mark Center Drive
   Alexandria, VA 22311

10. NMAWC..1
ATTN: Tom Little
32444 Echo Lane
BLDG 1/Suite 300
San Diego, CA 92147-5119

11. Surface Warfare Development Group 1
ATTN: Mr. Dave Gilbert, Technical Director
2200 Amphibious Drive
Norfolk, VA 23521-2896

12. Winford G. Ellis……………………………………………………………………….1
Chair of Undersea Warfare
Naval Postgraduate School
Monterey, CA 93943-5219

13. The Johns Hopkins University…………………………………………………...……1
Applied Physics Laboratory
11100 Johns Hopkins Road
Laurel, MD 20723-6099

14. Dr. W. Reynolds Monach…………………………………………………………..1
Daniel H. Wagner, Assoc.
2 Eaton Street, Suite 500
Hampton, VA 23669-4054